

Package ‘RATest’

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Type Package

Title Randomization Tests

Description A collection of randomization tests, data sets and examples. The current version focuses on three testing problems and their implementation in empirical work. First, it facilitates the empirical researcher to test for particular hypotheses, such as comparisons of means, medians, and variances from k populations using robust permutation tests, which asymptotic validity holds under very weak assumptions, while retaining the exact rejection probability in finite samples when the underlying distributions are identical. Second, the description and implementation of a permutation test for testing the continuity assumption of the baseline covariates in the sharp regression discontinuity design (RDD) as in Canay and Kamat (2017) <<https://goo.gl/UZFqt7>>. More specifically, it allows the user to select a set of covariates and test the aforementioned hypothesis using a permutation test based on the Cramer-von Miss test statistic. Graphical inspection of the empirical CDF and histograms for the variables of interest is also supported in the package. Third, it provides the practitioner with an effortless implementation of a permutation test based on the martingale decomposition of the empirical process for the goodness-of-fit testing problem with an estimated nuisance parameter. An application of this testing problem is the one of testing for heterogeneous treatment effects in a randomized control trial.

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BugReports <https://github.com/ignaciomsarmiento/RATest/issues>

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Collate 'CvM.stat.R' 'H.cdf.R' 'PT.Khmaladze.fit.R' 'RDperm.R' 'RPT.R'
'group.action.R' 'lee2008.R' 'plot.RDperm.R'
'summary.PT.Khmaladze.fit.R' 'summary.RDperm.R' 'summary.RPT.R'

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CvM.stat

Cramer - von Mises statistics

Description

Calculates the Cramer-von Mises test statistic

$$T(S_n) = \frac{1}{2q} \sum_{i=1}^{2q} (H_n^-(S_{n,i}) - H_n^+(S_{n,i}))^2$$

where $H_n^-(\cdot)$ and $H_n^+(\cdot)$ are the empirical CDFs of the the sample of baseline covariates close to the cutoff from the left and right, respectively. See equation (12) in Canay and Kamat (2017).

Usage

CvM.stat(Sn)

Arguments

Sn Numeric. The pooled sample of induced order statistics. The first column of S can be viewed as an independent sample of W conditional on Z being close to zero from the left. Similarly, the second column of S can be viewed as an independent sample of W conditional on Z being close to the cutoff from the right. See section 3 in Canay and Kamat (2017).

Value

Returns the numeric value of the Cramer - von Mises test statistic.

Author(s)

Mauricio Olivares Gonzalez

Ignacio Sarmiento Barbieri

References

Canay, I and Kamat V, (2017) Approximate Permutation Tests and Induced Order Statistics in the Regression Discontinuity Design. <http://faculty.wcas.northwestern.edu/~iac879/wp/RDDPermutations.pdf>

group.action

General Construction of Permutation Tests: Group Actions

Description

Calculates the pre-specified actions on data. Consider data Z taking values in a sample space Ω .

Let

$$\mathbf{G}$$

be a finite group of transformations from Ω onto itself, with

$$M = |\mathbf{G}|$$

. This function applies

$$gZ$$

as g varies in

$$\mathbf{G}$$

. If Z is a vector of size N and the actions g are permutations,

$$M = N!$$

. If the actions g are sign changes, then

$$M = \{1, -1\}^N$$

.

Usage

```
group.action(Z, M, type = "permutations")
```

Arguments

`Z` Numeric. A vector of size N to which the group action will act on. In the two-sample testing problem, Z is the pooled sample.

`M` Numeric. Number of actions to be performed. This is the number of transformations used in the stochastic approximation to the test. This is due to the fact that in some cases

$$M = |\mathbf{G}|$$

is too large, which makes the application of the actions computationally expensive.

`type` Character. The action to be performed. It represents

$$gx$$

, the action the action of

$$g \in \mathbf{G}$$

on

$$x \in \Omega$$

. It can be either permutations or sign changes.

Value

Numeric. A matrix of size

$$N \times M$$

where N is the size of input Z and M is the number of actions to be performed on Z .

Author(s)

Mauricio Olivares Gonzalez

Ignacio Sarmiento Barbieri

References

Lehmann, Erich L. and Romano, Joseph P (2005) Testing statistical hypotheses. Springer Science & Business Media.

Description

Calculates the empirical CDF of the sample of W conditional on Z being close to the cutoff from either the left or right. Given the induced order for the baseline covariates

$$W_{[q]}^-, W_{[q-1]}^-, \dots \leq W_{[1]}^-$$

or

$$W_{[1]}^+, W_{[2]}^+, \dots, W_{[q]}^+$$

, this function will calculate either

$$H_n^-(t) = \frac{1}{q} \sum_{i=1}^q I\{W_{[i]}^- \leq t\}$$

or

$$H_n^+(t) = \frac{1}{q} \sum_{i=1}^q I\{W_{[i]}^+ \leq t\}$$

depending on the argument of the function. See section 3 in Canay & Kamat (2017).

Usage

H.cdf(W, t)

Arguments

W Numeric. The sample of induced order statistics. The input can be either $\{W_{[q]}^-, W_{[q-1]}^-, \dots, W_{[1]}^-\}$ or $\{W_{[1]}^+, W_{[2]}^+, \dots, W_{[q]}^+\}$.

t Numeric. The scalar needed for the calculation of the CDF.

Value

Numeric. For a sample $W = (w_1, \dots, w_n)$, returns the fraction of observations less or equal to t .

Author(s)

Mauricio Olivares Gonzalez

Ignacio Sarmiento Barbieri

References

Canay, I and Kamat V, (2017) Approximate Permutation Tests and Induced Order Statistics in the Regression Discontinuity Design. <http://faculty.wcas.northwestern.edu/~iac879/wp/RDDPermutations.pdf>

 lee2008

Dataset used in Lee (2008)

Description

Randomized experiments from non-random selection in U.S. House elections

Format

A data frame with 6558 observations and two variables:

demsharenext Democrat vote share election t+1

difdemshare Running variable. Diff. democratic share

demshareprev Democrat vote share t-1

demwinprev Democrat win t-1

demofficeexp Democrat political experience t

othofficeexp Oppositions political experience t

demelectexp Democrat electoral experience t

othelectexp Opposition electoral experience t

Source

Mostly Harmless Econometrics Data Archive: <http://economics.mit.edu/faculty/angrist/data1/mhe>

References

Lee, D. (2008) Randomized experiments from non-random selection in U.S. House elections, *Journal of Econometrics*, 142, 675-697

 plot.RDperm

Plot RDperm

Description

Plots a histogram and empirical cdf

Usage

```
## S3 method for class 'RDperm'
plot(x, w, plot.class = "both", ...)
```

Arguments

x	Object of class "RDperm"
w	Character. Name of variable to be plotted
plot.class	Character. Can be: "both" for a histogram and cdf plot, "hist" for a histogram or "cdf" for only the cdf plot
...	Additional ggplot2 controls

Author(s)

Mauricio Olivares Gonzalez

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References

Canay, I and Kamat V, (2017) Approximate Permutation Tests and Induced Order Statistics in the Regression Discontinuity Design. <http://faculty.wcas.northwestern.edu/~iac879/wp/RDDPermutations.pdf>

Examples

```
## Not run:
permtest<-RDperm(W=c("demshareprev", "demwinprev"), z="difdemshare", data=lee2008)
plot(permtest, w="demshareprev")

## End(Not run)
```

PT.Khmaladze.fit *Non-Parametric Hypothesis Testing with a Nuisance Parameter: A Permutation Test*

Description

A permutation test of the two-sample goodness-of-fit hypothesis in the presence of an estimated nuisance parameter. The permutation test considered here is based on the Khmaladze transformation of the empirical process (Khmaladze (1981)), and adapted by Chung and Olivares-Gonzalez (2018).

Usage

```
PT.Khmaladze.fit(y1, y0, alpha = 0.05, n.perm = 999)
```

Arguments

y1	Numeric. A vector containing the response variable of the treatment group.
y0	Numeric. A vector containing the response variable of the control group.
alpha	Numeric. Nominal level for the test. The default is 0.05.
n.perm	Numeric. Number of permutations needed for the stochastic approximation of the p-values. The default is n.perm=999.

Value

An object of class "PT.Khmaladze.fit" is a list containing at least the following components:

n_populations	Number of grups.
N	Sample Size.
T.obs	Observed test statistic.
shift	The estimated nuisance parameter (average treatment effect).
cv	Critical Value. This value is used in the general construction of a randomization test.
pvalue	P-value.
T.perm	Vector. Test statistic recalculated for all permutations used in the stochastic approximation.
n_perm	Number of permutations.
sample_sizes	Groups size.

Author(s)

Mauricio Olivares-Gonzalez

Ignacio Sarmiento Barbieri

References

Khmaladze, E. (1981). Martingale Approach in the Theory of Goodness-of-fit Tests. *Theory of Probability and its Application*, 26: 240–257. Chung, Eunyi and Mauricio Olivares (2019). Non-Parametric Hypothesis Testing with a Nuisance Parameter: A Permutation Test Approach. Working Paper.

Examples

```
## Not run:
Y0 <- rnorm(100, 1, 1)
# Treatment Group with constant shift equals to 1
Y1 <- Y0 + 1
Tx = sample(100) <= 0.5*(100)
# Observed Outcome
Y = ifelse( Tx, Y1, Y0 )
dta <- data.frame(Y = Y, Z = as.numeric(Tx))
pt.GoF<-PT.Khmaladze.fit (dta$Y[dta$Z==1],dta$Y[dta$Z==0],n.perm = 49)
summary(pt.GoF)

## End(Not run)
```


RDperm

*Regression Discontinuity Design Permutation Test***Description**

A permutation test for continuity of covariates in Sharp Regression Discontinuity Design as described in Canay and Kamat (2017).

Usage

```
RDperm(W, z, data, n.perm = 499, q_type = 10, cutoff = 0,
       test.statistic = "CvM")
```

Arguments

<code>W</code>	Character. Vector of covariates names. The procedure will test the null hypothesis of continuity of the distribution of each element in <code>W</code> at the cutoff.
<code>z</code>	Character. Running variable name. This is the scalar random variable that defines, along with the cutoff, the treatment assignment rule in the sharp regression discontinuity design.
<code>data</code>	Data.frame.
<code>n.perm</code>	Numeric. Number of permutations needed for the stochastic approximation of the p-values. See remark 3.2 in Canay and Kamat (2017). The default is <code>B=499</code> .
<code>q_type</code>	A fixed and small (relative to the sample size) natural number that will define the q closest values of the order statistic of Z to the right and to the left of the cutoff. The default, 'rot', value is given by the feasible rule of thumb in footnote 4 of Canay and Kamat (2017), section 3.1. If 'arot', it calls for the Rule of Thumb described in equation (15) of Canay and Kamat (2017), section 3.1. The default option grows at a slower rate than the optional rule of thumb, but adds a larger constant.
<code>cutoff</code>	Numeric. The scalar defining the threshold of the running variable.
<code>test.statistic</code>	Character. A rank test statistic satisfying rank invariance. The default is a Cramer-von Mises test statistic.

Value

The functions `summary` and `plot` are used to obtain and print a summary and plot of the estimated regression discontinuity. The object of class `RDperm` is a list containing the following components:

<code>results</code>	Matrix. Test Statistic, P-values and Q
<code>test.statistic</code>	Test Statistic
<code>q_type</code>	Type of Q used in the calculations, can be either, "Defined by User", the "Rule of Thumb" or the "Alternative Rule of Thumb".

n_perm	number of permutations
rv	Character. Running variable name
Z	Vector. Running Variable
cutoff	cutoff
data	data set
S	Matrix. Pooled sample of induced order statistics
S_perm	List. Permutations of the induced order statistic.

Author(s)

Mauricio Olivares Gonzalez

Ignacio Sarmiento Barbieri

References

Canay, I and Kamat V, (2017) Approximate Permutation Tests and Induced Order Statistics in the Regression Discontinuity Design. <http://faculty.wcas.northwestern.edu/~iac879/wp/RDDPermutations.pdf>

Examples

```
permtest<-RDperm(W=c("demshareprev"), z="difdemshare", data=lee2008)
summary(permtest)
## Not run:
permtest<-RDperm(W=c("demshareprev", "demwinprev"), z="difdemshare", data=lee2008)
summary(permtest)

## End(Not run)
```

RPT

Robust Permutation Test

Description

This function considers the k-sample problem of comparing general parameters, such as means, medians, or parameters that depend on the joint distribution using permutation tests. Under weak assumptions for comparing estimator, the permutation tests implemented here provide a general test procedure whereby the asymptotic validity of the permutation test holds while retaining the exact rejection probability α in finite samples when the underlying distributions are identical. Here we will consider three test for the 2 sample case, but the function works for k-samples.

Difference of means: Here, the null hypothesis is of the form $H_0 : \mu(P) - \mu(Q) = 0$, and the corresponding test statistic is given by

$$T_{m,n} = \frac{N^{1/2}(\bar{X}_m - \bar{Y}_n)}{\sqrt{\frac{N}{m}\sigma_m^2(X_1, \dots, X_m) + \frac{N}{n}\sigma_n^2(Y_1, \dots, Y_n)}}$$

where \bar{X}_m and \bar{Y}_n are the sample means from population P and population Q , respectively, and $\sigma_m^2(X_1, \dots, X_m)$ is a consistent estimator of $\sigma^2(P)$ when X_1, \dots, X_m are i.i.d. from P . Assume consistency also under Q .

Difference of medians: Let F and G be the CDFs corresponding to P and Q , and denote $\theta(F)$ the median of F i.e. $\theta(F) = \inf\{x : F(x) \geq 1/2\}$. Assume that F is continuously differentiable at $\theta(P)$ with derivative F' (and the same with F replaced by G). Here, the null hypothesis is of the form $H_0 : \theta(P) - \theta(Q) = 0$, and the corresponding test statistic is given by

$$T_{m,n} = \frac{N^{1/2} \left(\theta(\hat{P}_m) - \theta(\hat{Q}) \right)}{\hat{v}_{m,n}}$$

where $\hat{v}_{m,n}$ is a consistent estimator of $v(P, Q)$:

$$v(P, Q) = \frac{1}{\lambda} \frac{1}{4(F'(\theta))^2} + \frac{1}{1-\lambda} \frac{1}{4(G'(\theta))^2}$$

Choices of $\hat{v}_{m,n}$ may include the kernel estimator of Devroye and Wagner (1980), the bootstrap estimator of Efron (1992), or the smoothed bootstrap Hall et al. (1989) to list a few. For further details, see Chung and Romano (2013). Current implementation uses the bootstrap estimator of Efron (1992)

Difference of variances: Here, the null hypothesis is of the form $H_0 : \sigma^2(P) - \sigma^2(Q) = 0$, and the corresponding test statistic is given by

$$T_{m,n} = \frac{N^{1/2} (\hat{\sigma}_m^2(X_1, \dots, X_m) - \hat{\sigma}_n^2(Y_1, \dots, Y_n))}{\sqrt{\frac{N}{m} (\hat{\mu}_{4,x} - \frac{(m-3)}{(m-1)} (\hat{\sigma}_m^2)^2) + \frac{N}{n} (\hat{\mu}_{4,y} - \frac{(n-3)}{(n-1)} (\hat{\sigma}_y^2)^2)}}$$

where $\hat{\mu}_{4,m}$ the sample analog of $E(X - \mu)^4$ based on an iid sample X_1, \dots, X_m from P . Similarly for $\hat{\mu}_{4,n}$.

We could also have the case when the parameter of interest is a function of the joint distribution. The examples considered here are

Lehmann (1951) two-sample U statistics: Consider testing $H_0 : P = Q$, or the more general hypothesis that P and Q only differ in location against the alternative that the Y 's are more spread out than the X 's. The null hypothesis is of the form

$$H_0 : P(|Y - Y'| > |X - X'|) = 1/2$$

.

Two-sample Wilcoxon statistic, where the null hypothesis is of the form

$$H_0 : P(X \leq Y) = 1/2$$

.

Two-sample Wilcoxon statistic without continuity assumption. In this case, the null hypothesis is of the form

$$H_0 : P(X \leq Y) = P(Y \leq X)$$

.

Hollander (1967) two-sample U statistics. The null hypothesis is of the form

$$H_0 : P(X + X' < Y + Y') = 1/2$$

.

Usage

```
RPT(formula, data, test = "means", n.perm = 499, na.action,
    wilcoxon.option = "continuity")
```

Arguments

<code>formula</code>	a formula object, with the response on the left of a <code>~</code> operator, and the groups on the right.
<code>data</code>	a <code>data.frame</code> in which to interpret the variables named in the formula. If this is missing, then the variables in the formula should be on the search list.
<code>test</code>	test to be performed. Multiple options are available, depending on the nature of the testing problem. In general, we have two types of problem. First, when the researcher is interested in comparing parameters. In this case, "means" will perform a Difference of Means, "medians" a Difference of Medians, "variances" a Difference of Variances. This case allows for 2 or more population comparisons. For the test of difference of medians the Efron (1992) bootstrap estimator is used to estimate the variances (for further details, see Chung and Romano (2013)). Second, when the parameter of interest is a function of the joint distribution. In this case, "lehmann.2S.test" will perform Lehmann (1951) two-sample U statistics, "wilcoxon.2s.test" the two-sample Wilcoxon test (with or without continuity assumption), and "hollander.2S.test" Hollander (1967) two sample U statistics. In this case, only 2 sample comparisons are permitted.
<code>n.perm</code>	Numeric. Number of permutations needed for the stochastic approximation of the p-values. See remark 3.2 in Canay and Kamat (2017). The default is <code>n.perm=499</code> .
<code>na.action</code>	a function to filter missing data. This is applied to the <code>model.frame</code> . The default is <code>na.omit</code> , which deletes observations that contain one or more missing values.
<code>wilcoxon.option</code>	Continuity assumption for Wilcoxon test" with continuity ("continuity") or without ("discontinuity"). The default is "continuity"

Value

An object of class "RPT" is a list containing at least the following components:

<code>description</code>	Type of test, can be Difference of Means, Medians, or Variances.
<code>n_populations</code>	Number of groups.
<code>N</code>	Sample Size.
<code>T.obs</code>	Observed test statistic.
<code>pvalue</code>	P-value.
<code>T.perm</code>	Vector. Test statistics from the permutations.
<code>n_perm</code>	Number of permutations.
<code>parameters</code>	Estimated parameters.
<code>sample_sizes</code>	Groups lengths.

Author(s)

Mauricio Olivares Gonzalez
 Ignacio Sarmiento Barbieri

References

Chung, E. and Romano, J. P. (2013). Exact and asymptotically robust permutation tests. *The Annals of Statistics*, 41(2):484–507. Chung, E. and Romano, J. P. (2016). Asymptotically valid and exact permutation tests based on two-sample u-statistics. *Journal of Statistical Planning and Inference*, 168:97–105. Devroye, L. P. and Wagner, T. J. (1980). The strong uniform consistency of kernel density estimates. In *Multivariate Analysis V: Proceedings of the fifth International Symposium on Multivariate Analysis*, volume 5, pages 59–77. Efron, B. (1992). Bootstrap methods: another look at the jackknife. In *Breakthroughs in statistics*, pages 569–593. Springer. Hall, P., DiCiccio, T. J., and Romano, J. P. (1989). On smoothing and the bootstrap. *The Annals of Statistics*, pages 692–704. Hollander, M. (1967). Asymptotic efficiency of two nonparametric competitors of wilcoxon's two sample test. *Journal of the American Statistical Association*, 62(319):939–949. Lehmann, E. L. (1951). Consistency and unbiasedness of certain nonparametric tests. *The Annals of Mathematical Statistics*, pages 165–179.

Examples

```
## Not run:
male<-rnorm(50,1,1)
female<-rnorm(50,1,2)
dta<-data.frame(group=c(rep(1,50),rep(2,50)),outcome=c(male,female))
rpt.var<-RPT(dta$outcome~dta$group,test="variances")
summary(rpt.var)

## End(Not run)
```

```
summary.PT.Khmaladze.fit
```

*Summarizing Permutation Test for Heterogeneous Treatment Effects
 with Estimated Nuisance Parameter*

Description

```
summary method for class "PT.Khmaladze.fit"
```

Usage

```
## S3 method for class 'PT.Khmaladze.fit'
summary(object, ..., digits = max(3,
  getOption("digits") - 3))
```

Arguments

object	an object of class "PT.Khmaladze.fit", usually a result of a call to PT.Khmaladze.fit
...	unused
digits	number of digits to display

Value

summary.PT.Khmaladze.fit returns an object of class "summary.PT.Khmaladze.fit" which has the following components

results	Matrix with the Testing Problem, Sample Sizes, Number of Permutations, ATE, Test Statistic, Critical value and P-value.
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Author(s)

Mauricio Olivares Gonzalez
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summary.RDperm	<i>Summarizing Regression Discontinuity Design Permutation Test</i>
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Description

summary method for class "RDPerm"

Usage

```
## S3 method for class 'RDperm'
summary(object, digits = max(3, getOption("digits") -
  3), ...)
```

Arguments

object	an object of class "RDperm", usually a result of a call to RDperm
digits	number of digits to display
...	unused

Value

summary.RDperm returns an object of class "summary.RDperm" which has the following components

results	Matrix with the Test Statistic, P-values and Q used
---------	---

Author(s)

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`summary.RPT`*Summarizing Robust Permutation Test*

Description

summary method for class "RPT"

Usage

```
## S3 method for class 'RPT'  
summary(object, ..., digits = max(3, getOption("digits") -  
  3))
```

Arguments

<code>object</code>	an object of class "RPT", usually a result of a call to RPT
<code>...</code>	unused
<code>digits</code>	number of digits to display

Value

`summary.RPT` returns an object of class "summary.RPT" which has the following components

<code>results</code>	Matrix with the Testing Problem, Point Estimates, Sample Sizes, Test Statistic, P-values and Sample Sizes.
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Author(s)

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